# First Order Logic Worksheet

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## Converting to CNF and DNF

Convert the following sentences to conjunctive normal form and disjunctive normal form.

1. $(A \to B) \to C$
2. $A \to (B \to C)$
3. $(A \to B) \vee (B \to A)$
4. $(\neg P \to (P \to Q))$
5. $(P \to (Q \to R)) \to (P \to (R \to Q))$
6. $(P \to Q) \to ((Q \to R) \to (P \to R))$

## First Order Logic Sentences

For each of the following English sentences, write a corresponding sentence in FOL.

1. The only good extraterrestrial is a drunk extraterrestrial.
2. The Barber of Seville shaves all men who do not shave themselves.
3. There are at least two mountains in England.
4. There is exactly one coin in the box.
5. There are exactly two coints in the box.
6. The largest coin in the box is a quarter.
7. No mountain is higher than itself.
8. All students get good grades if they study.

## FOL Interpretations, Part 1

For each group of sentences, write an interpretation under which the last sentence is false and all the rest are true.

1. $\forall x(Hx \to Gx)$
   $\forall x(Fx \to Gx)$
   $\exists x(Fx \wedge Hx)$
2. $\forall x \exists y Rxy$
   $\exists y \forall x Fxy$
3. $\forall x(Fx \to Ga)$
   $\forall x Fx \to Ga$

## FOL Interpretations, Part 2

For each group of sentences, give an interpretation in which all sentences are true.

1. $(\forall x Px \vee Qx) \rightarrow \exists x Rx$
   $\forall x Rx \rightarrow Qx$
   $\exists x Px \wedge \neg Qx$
2. $\forall x \neg Fxx$
   $\forall x \forall y \forall z (Fxy \wedge Fyz \rightarrow Fxz)$
   $\forall x \exists y Fxy$
3. $\forall x \exists y Fxy$
   $\forall x (Gx \rightarrow \exists y Fyx)$
   $\exists x Gx$
   $\forall x \neg Fxx$

## FOL Semantics

(**6**) Consider a world with objects **a**, **b**, and **c**. We'll look at a logical languge with constant symbols $X$, $Y$, and $Z$, function symbols $f$ and $g$, and predicate symbols $p$, $q$, and $r$.
Note: This notation scheme is slightly different than what we've seen. But good to see some notational variants. Consider the following interpretation:

- $I(X) = \mathbf{A}$, $I(Y) = \mathbf{A}$, $I(Z) = \mathbf{B}$
- $I(f) = \{\langle \mathbf{A}, \mathbf{B} \rangle, \langle \mathbf{B}, \mathbf{C} \rangle, \langle \mathbf{C}, \mathbf{C} \rangle\}$
- $I(p) = \{\mathbf{A}, \mathbf{B}\}$
- $I(q) = \{\mathbf{C}\}$
- $I(r) = \{\langle \mathbf{B}, \mathbf{A} \rangle, \langle \mathbf{C}, \mathbf{B} \rangle, \langle \mathbf{C}, \mathbf{C} \rangle\}$

For each of the following sentences, say whether it is true or false in the given interpretation $I$:

1. $q(f(Z))$
2. $r(X, Y)$
3. $\exists w.f(w) = Y$
4. $\forall w.r(f(w), w)$
5. $\forall u, v.r(u, v) \rightarrow (\forall w.r(u, w) \rightarrow v = w)$
6. $\forall u, v.r(u, v) \rightarrow (\forall w.r(w, v) \rightarrow u = w)$