Chapter 30 Solutions

New Beginnings Theory 2018

David Lu

August 21, 2018

Here are some selected solutions to the interpretation practice exercises from chapter 30. Remember that there are many (infinitely many) correct solutions to these exercises and here is only one solution.

Avoid relying on these solutions. There's a difference between looking at a solution and recognizing that it's correct and coming up with a solution completely on your own. If rely too much on looking at the solution to help you get started, you won't develop the problem solving skills that are necessary to solve the problems on your own.

If you'd like more examples of interpretations, you the book's solutions for chapter 31, parts H and I, contain some more.

(Part A) Contingency

1. $Da \land Db$

True:
$D = \{a, b\}$
Da, Db
False:
$D = \{a, b\}$
$\neg Da, Db$

3. $Pm \land \neg \forall x Px$

True:
$D = \{m, n\}$
$Pm, \neg Pn$
False:
$D = \{m\}$
$\neg Pm$

5. $\forall x(Wxmn \lor \exists yLxy)$

True:
$D = \{m, n\}$
Predicates that are true:
Lmm, Lnn
All others are false.

False: $D = \{m, n\}$
Predicates are false of everything.
i.e. $(\neg Wmmn, \neg Wnmn, \neg Wmmm, \neg Wnnn, \neg Wmnm, \neg Wmnn, \neg Lmm, \neg Lnn, \neg Lmn, \neg Lnm)$

7. $\exists x(x = h \land x = i)$

True:
$D = \{h, i\}$
$h = i$

False:
$D = \{h, i\}$
$\neg h = i$

Part C : Consistency

1. $Ma, \neg Na, Pa, \neg Qa$

Consistent:
$D = \{a\}$
$Ma, \neg Na, Pa, \neg Qa$

3. $\neg(Ma \land \exists Ax), Ma \lor Fa, \forall x(Fx \rightarrow Ax)$

Consistent:
$D = \{a\}$
$\neg Ma, Fa, Aa$

5. $\forall yGy, \forall x(Gx \rightarrow Hx), \exists y \neg Iy$

Consistent:
$D = \{a\}$
$Ga, Ha, \neg Ia$

7. $\exists xXx, \exists xYx, \forall x(Xx \leftrightarrow \neg Yx)$

Consistent:
$D = \{a, b\}$
$Xa, \neg Ya, \neg Xb, Yb$

9. $\exists z(Nz \land Ozz), \forall x\forall y(Oxy \rightarrow Oyx)$

Consistent:
$D = \{a\}$
Na, Oaa

11. $\neg Raa, \forall x(x = a \lor Rxa)$

Consistent:
$D = \{a, b\}$
$\neg Raa, Rba, Rbb, Rab, \neg a = b$ (everything is identical to itself, so not necessary state explicitly)

13. $\exists x \exists y((Zx \land Zy) \land x = y), \neg Zd, d = e$

Consistent:
$D = \{a, d, e\}$
$Za, \neg Zd, \neg Ze, \neg a = d, \neg a = e, d = e$

Part D : Invalidity

1. $\forall x(Ax \to Bx) \therefore \exists x Bx$

Invalid:
$D = \{a\}$
$\neg Aa, \neg Ba$

3. $\exists x(Px \to Qx), \therefore \exists x Px$

Invalid:
$D = \{a\}$
$\neg Pa, \neg Qa$

5. $Rde, \exists x Rxd \therefore Red$

Invalid:
$D = \{a, d, e\}$
$Rad, Rde, \neg Red, Raa, Rae, Rdd, Rda, Ree, Rea$

7. $\forall x Oxc, \forall x Ocx \therefore \forall x Oxx$

Invalid:
$D = \{a, c\}$
$Oac, Oca, \neg Oaa, Occ$

9. $Lab \to \forall x Lxb, \exists x Lxb \therefore Lbb$

Invalid:
$D = \{a, b, c\}$
$Lcb, \neg Lbb, \neg Laa, \neg Lab, \neg Lac$
$\neg Lbc, \neg Lba, \neg Lcc, \neg Lca$