Final Exam

CS 251, Winter 2018
Discrete Structures II
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Name ________________________________

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This is a closed-book (no notes, cellphones, smart watches, etc...) exam. Show your work for partial credit. Read the questions carefully. Write legibly and document your proof steps. Feel free to use the back of each page if you need more room for work.

Part 1: True/False (3 pts each; 15 pts total)

True False A) Resolution can be used to show that an argument is valid.

True False B) Two interpretations are necessary to show that two sentences are logically equivalent.

True False C) All subproofs must be discharged before a proof can be considered complete.

True False D) $\forall x \forall y((Ax \land Ay) \to Rxy)$ and $\forall y \forall x((Ax \land Ay) \to Rxy)$ are logically equivalent.

True False E) If a and b are identical and Pa is true, then Pb is true.

Part 2: Translations Translate the sentences to FOL (5 pts each; 15 pts total)
A) Everyone who studies Computer Science has a friend who can program.
(Cx: x studies computer science, Fxy: x is y's friend, Px: x can program)

B) If there are at least three tarantulas in the car, then I'm not getting in the car.
(Tx: x is a tarantula, Cx: x is in the car, Gx: x is getting in the car.)

C) Bob doesn't love himself, but someone loves him.
(Lxy: x loves y)

Part 3: Tautology, Contingency, Contradiction Determine whether the formula is a *tautology*, *contingency*, or *contradiction*, then demonstrate your answer with the appropriate method (10 pts each; 30 pts total)

A) $\forall x((Px \wedge Qx) \rightarrow Rx) \rightarrow [(\forall x Px \wedge \forall x Qx) \rightarrow \forall x Rx]$

B) $\forall x \exists y Rxy \leftrightarrow \forall x \exists y Ryx$

C) $\forall x \forall y((Px \wedge Py) \rightarrow x = y)$

Part 4: Validity and Invalidity Determine if the following arguments are valid or not. Demonstrate with appropriate method. (15 pts each; 30 pts total)

A) $\neg\exists x(Px \wedge Qx), \exists x Px \therefore \neg\forall x(Px \rightarrow Qx)$

B) $\exists x\forall y Pxy$, $\forall x(Pxx \rightarrow \exists y Qyx))$, $\therefore \exists y\exists x Qxy$ (Hint: This is valid)

Part 5: Resolution (10 pts)

A) Demonstrate that the following argument is valid using resolution.
$A \rightarrow (B \rightarrow C) \therefore (A \wedge B) \rightarrow C$